Benha University
Faculty of Engineering at Shoubra
Elec. Engineering Department
2^{nd} Year Elec.
BAS211 Numerical Analysis

كلية الهندسة بشبرا
FACULTY OF ENGINEERING AT SHOUBRA

Midterm Exam (1)
No. of Questions: 3
Total Marks: 25
Date: 12 / 11 / 2025
Duration: 1 hour

اسم الطالب:	رقم السكشن:	المسلسل:

Instructions:
- Answer all questions.
- Show all necessary steps to receive full mark.
- Use appropriate numerical approximations up to 4 decimal places where applicable.

Question 1 [10 marks]

(1.a) Using the **fixed-point** iterative method to approximate the root of the equation $x^3 - x - 1 = 0$, and stop when $| x_{n+1} - x_n | < 10^{-4}$.

Solution:

$$x = (x+1)^{1/3}$$

Start with $x_0 = 1.5$ (midpoint of $[1,2]$)

Iterations

i	x_i	$\| x_{n+1} - x_n \|$
0	1.5	
1	1.3572088083	0.14279
2	1.3308609588	0.02635
3	1.3258837742	0.00498
4	1.3249393634	0.00094
5	1.3247600113	0.00017
6	1.3247259452	$0.000034 < 10^{-4}$

So the iteration stopped at **iteration 6** (i.e. after computing x_6), and the approximation is

$$\boxed{x \approx 1.3247259452.}$$

(1.b) Use **Gauss-Seidel** Iterative method to find the solution of:

$$20x + y - 2z = 17$$
$$2x - 3y + 20z = 25$$
$$3x + 20y - z = -18.$$

Solution:

$$x_{n+1} = \frac{1}{20}(17 - y_n + 2z_n)$$
$$y_{n+1} = \frac{1}{20}(-18 - x_{n+1} - 20z_n)$$
$$z_{n+1} = \frac{1}{20}(25 - 2x_{n+1} + 3y_{n+1})$$

Iteration	(x)	(y)	(z)
0	0.0000	0.0000	0.0000
1	0.8500	-1.0275	1.01087
2	1.0024	-0.9998	0.9997
3	0.9999	-1.0000	1.0000
4	1	-1	1

Final approximate solution (after 4 iterations):

$$x = 1, \qquad y = -1, \qquad z = 1$$

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Question 2 [7 marks]

(2.a) Find the missing term in the following table using **Newton divided difference** formula.

x	0	1	2	3	4
y	1	3	9		81

Solution:

x	**y**	$f[x_0]$	$f[x_0,x_1]$	$f[x_0,x_1,x_2]$
0	**1**			
		$\frac{3-1}{1-0}=2$		
1	**3**		$\frac{6-2}{2-0}=2$	
		$\frac{9-3}{2-1}=6$		$\frac{10-2}{4-0}=2$
2	**9**		$\frac{36-6}{4-1}=10$	
		$\frac{81-9}{4-2}=36$		
4	**81**			

Substitute the computed values:

$$\boldsymbol{P(x) = 1 + 2(x-0) + 2(x-0)(x-1)}$$
$$\boldsymbol{+2(x-0)(x-1)(x-2)}$$
$$\boldsymbol{P(3) = 1 + 2(3) + 2(3)(3-1)}$$
$$\boldsymbol{+\,2(3)(3-1)(3-2) = 31}$$

Then the missing term is 31

x	0	1	2	3	4
y	1	3	9	**31**	81

(2.b) Calculate fitting exponential equation

$$\boldsymbol{y = ae^{-bx^2}}$$

using the **Least squares** method

x	1	2	3	4	5	8
y	9.01	6.01	6.07	2.02	0.22	0.02

Let $Y = ln\,y, X = x^2$

Then the model becomes a **linear regression** form:

$$Y = A + BX,$$

where $A = \ln a\,,\ B = -b.$

Then

$$\sum Y = nA + B\sum X,$$
$$\sum XY = A\sum X + B\sum X^2,$$

x	y	$X = x^2$	$(Y = 1/ln\,y)$	XY	X^2
1	9.01	1	2.198		
2	6.01	4	1.794		
3	6.07	9	1.805		
4	2.02	16	0.703		
5	0.22	25	-1.514		
8	0.02	64	-3.912		
		119	1.074	-251.351	5075

$A = 2.1697$, and $\boldsymbol{B} = -0.1004$

And,

$$\mathrm{a = e^{A} = e^{2.1697} = 8.752}$$
$$b = -B = 0.1004$$

Final fitted exponential equation:

$$\boxed{y = 8.752\, e^{-0.1004\, x^2}}$$

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Question 3 [8 marks]

(3.a) Consider the following table of data:

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	0.9799	0.9178	0.8080	0.6386	0.3844

Use the best accuracy formulas to approximate $f'(0.6)$ and $f''(0.6)$.

Solution:

First derivative

$$f'(x) = \frac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)}{12\,h}$$

Second derivative

$$f''(x) = \frac{1}{12\,h^2}[-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)]$$

So

$$f'(0.6) \approx -0.68254$$

$$f''(0.6) \approx -1.46229$$

(3.b) Evaluate

$$\int_0^1 \frac{e^x}{\sqrt{x}}\,dx$$

using **Simpson's rule** at $h = 0.1$.

Solution:

The integrand is singular at $x = 0$, so first remove the singularity with the substitution

$$x = t^2 \qquad (t = \sqrt{x}), \qquad dx = 2t\,dt.$$

Then

$$\mathrm{I} = \int_0^1 \frac{e^x}{\sqrt{x}}\,dx = 2\int_0^1 e^{t^2}\,dt,$$

and the integrand e^{t^2} is smooth on [0,1].

Apply Simpson's rule on the t-integral with step

$h = 0.1$ (so $n = 10$subintervals):

$$\int_0^1 e^{t^2}\,dt \approx \frac{h}{3}[f(0) + f(1) + 4\sum_{i\text{ odd}} f(t_i) + 2\sum_{i\text{ even}} f(t_i)],$$

with $f(t) = e^{t^2}$and $t_i = 0.1i$. Multiplying the Simpson result by 2 gives the desired integral.

Numerical result

$$\int_0^1 \frac{e^x}{\sqrt{x}}\,dx \approx 2.9254$$

Best Wishes

Dr. Ayman Fayez